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On the possible running of the cosmological “constant”

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ABSTRACT

Despite the many outstanding cosmological observations leading to a strong evidence for a non-vanishing cosmological constant (CC) term Λ in the gravitational field equations, the theoretical status of this quantity seems to be lagging well behind the observational successes. It thus seems timely to revisit some fundamental aspects of the CC term in Quantum Field Theory (QFT). We emphasize that, in curved space–time, nothing a priori prevents this term from potentially having a mild running behavior associated to quantum effects. Remarkably, this could be the very origin of the dynamical nature of the Dark Energy, in contrast to many other popular options considered in the literature. In discussing this possibility, we also address some recent criticisms concerning the possibility of such running. Our conclusion is that, while there is no comprehensive proof of the CC running, there is no proof of the non-running either. The problem can be solved only through a deeper understanding of the vacuum contributions of massive quantum fields on a curved space–time background. We suggest that such investigations are at the heart of one of the most important endeavors of fundamental theoretical cosmology in the years to come.

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1. Introduction

The relevance of the CC problems [1,2] has triggered a renewed interest on the dynamical quantum effects on the vacuum energy density and their possible implications in cosmology. The standard way to parameterize the leading quantum effects is the renormalization group (RG). The theoretical background for the RG running of the CC has been established in the papers [3–5] where this running was considered in the framework of Quantum Field Theory (QFT) in curved space–time, and also in [6] where the underlying theory was Quantum Gravity (QG). From the viewpoint of the physical interpretation of the RG, one can identify three approaches dealing with the CC problem: one of them can be described as an attempt to solve the “old” CC problem [1] via the RG screening at low energies [7–11]. Another is the functional renormalization group approach to QG [12,13], which is based on the Wilsonian notion of average action and its RG equation; it is applied in the framework of QG and is formulated non-perturbatively [14]. Finally, there is the approach proposed by the present authors [15,16], in which it is explored the possibility of having a relatively moderate running of the CC within perturbative QFT in a curved background [2]. The latter kind of approach is closer to the methods of Particle Physics phenomenology. In this same line, we have the contributions [17–23]. In all these cases, one studies

the running of the CC without making direct reference to the old CC problem [1]. Let us also point out that most of the aforesaid RG papers use the Minimal Subtraction (MS) scheme of renormalization, which is a pretty well established albeit not directly physical procedure, whereas a few recent works have dealt with the yet undeveloped (despite badly needed) physical scheme of renormalization in cosmology. This scheme, when a rigorous formulation becomes finally available, should appropriately extend the existing physical renormalization schemes [24] into curved space–time.

In the papers [15,16], we suggested the possibility of the CC running, although this cannot be rigorously proven in the context of the present day QFT in curved space. Recently, an attempt to prove that such running is mathematically impossible was undertaken in [25]. However, the argumentation presented in that work is misleading. This conclusion will ensue here as a corollary of our general discussion (see also [26] for more details).¹ Our main aim in this Letter is of general nature; we wish to critically assess the physical conditions leading to the CC running in QFT, and also to emphasize that this issue could be of paramount importance for the theoretical cosmology. This is especially so after the many efforts devoted during the last quarter of a century trying to unsuccessfully replace Einstein’s CC with a variety of ersatz entities of different nature without, unfortunately, improving significantly the overall status of the difficult CC problem(s) [1].

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The Letter is organized as follows. In Section 2 we briefly discuss the CC and the CC problem. In particular, we demonstrate the importance of the curved-space metric and remember that in the conformal parametrization the Einstein–Hilbert action with CC is equivalent to the scalar field action and that the renormalization of the CC term is analogous to the one of the scalar potential term in the Nambu–Jona–Lasinio (NJL) model. In Section 3 we briefly review the notion of running in QFT. Section 4 is devoted to the detailed analysis of the criticism of our previous papers in [25]. In Section 5 we discuss the unusual possible form of the effective action of the metric which can be responsible for the running and show that, despite very strong restrictions, such running cannot be ruled out. Finally, in Section 6 we draw our conclusions.

2. What is Λ , and what are the Λ problems

Before starting to discuss the CC running, it is worthwhile to remember what is the CC term Λ and what is the RG running. The modern gravitational physics starts from the action of the form

$$S_{\text{total}} = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} (R + 2\Lambda) + S_{\text{HD}} + S_{\text{mater}}. \quad (1)$$

Here the first term is the Einstein–Hilbert action with the cosmological constant Λ ; the second term S_{HD} includes higher derivatives (cf. Section 5), which are necessary for the consistency of a quantum theory in curved space (see, e.g., [29,30] or [31] for an introduction) and the last term, S_{mater} , represents the action of matter, responsible for the energy–momentum tensor T_{μ}^{ν} . In the low-energy domain, one can in principle disregard S_{HD} and the dynamical equations for the metric take on the Einstein form

$$R_{\mu}^{\nu} - \frac{1}{2} R \delta_{\mu}^{\nu} = 8\pi G T_{\mu}^{\nu} + \Lambda \delta_{\mu}^{\nu}. \quad (2)$$

If we consider matter as an isotropic fluid with energy density ρ and pressure p , the energy–momentum tensor has the following form (in the locally co-moving frame):

$$T_{\mu}^{\nu} = \text{diag}(\rho, -p, -p, -p). \quad (3)$$

It is easy to see that the Λ -dependent term in (2) has exactly the form (3), with the “vacuum energy density” $\rho_{\Lambda}^{\text{vac}}$ and “vacuum pressure” p_{Λ}^{vac} being

$$\rho_{\Lambda}^{\text{vac}} = \frac{\Lambda}{8\pi G} = -p_{\Lambda}^{\text{vac}}. \quad (4)$$

Thus, the vacuum part of the energy–momentum tensor is $(T^{\text{vac}})_{\mu}^{\nu} = \rho_{\Lambda}^{\text{vac}} \delta_{\mu}^{\nu}$, which justifies to call (4) the “vacuum energy”.

Let us start the discussion of the CC term by making an important remark. Without gravity the CC term is nothing but an irrelevant constant. The CC acquires dynamical significance only through the Einstein equations (2), which tell us the space–time is curved. To better understand the role of the curved space for the CC term, let us consider another parametrization of the metric

$$g_{\mu\nu} = \frac{\chi^2}{M_p^2} \bar{g}_{\mu\nu}, \quad (5)$$

where $\bar{g}_{\mu\nu}$ is some fiducial metric with fixed nonzero determinant; for instance, it can be the flat metric $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$. Furthermore, $\chi = \chi(x)$ is a scalar field which can be identified as a conformal factor of the metric, and $M_p \equiv G^{-1/2}$ is the Planck mass. It is easy to see that the CC term looks rather different in these new variables:

$$S_{\Lambda} = - \int d^4x \sqrt{-g} \frac{\Lambda}{8\pi G} = - \int d^4x \sqrt{-\bar{g}} f \chi^4, \quad (6)$$

where $f = \frac{\Lambda}{8\pi G M_p^4} = \frac{\Lambda}{8\pi M_p^4}$.

The last expression is nothing but the usual quartic term in the potential for the scalar interaction. One may note that, under the same change of variables, the Einstein–Hilbert term transforms into the action of the scalar field χ with the negative kinetic term and non-minimal conformal coupling to curvature [32–34]. Furthermore, the massive term in the spinor Lagrangian becomes a Yukawa-type interaction between the fermion and the scalar degree of freedom of the metric χ .

One important consequence which follows from the above observation is that the renormalization of the Newton constant and the CC due to the quantum effects of the spinor field is completely similar to the renormalization in the well-known NJL model. In this model, one does not quantize the scalar field, exactly as we do not intend to quantize the metric in QFT in curved space–time. Furthermore, within the MS scheme of renormalization, the RG running of the CC in the theory of spinor field is mathematically equivalent to the one of the parameter f in the NJL model [35]. Similar equivalence can be easily established for the quantum effects of a free massive scalar field φ – in this case, the quantum field interacts with χ via the $\varphi^2 \chi^2$ -term.

On the phenomenological side, the CC is the most natural candidate for the Dark Energy (DE) which is responsible for the cosmic acceleration. However, all models of the DE must face the so-called “old CC problem” [1], i.e. the formidable task of trying to understand the enormous ratio $r = \rho_{\Lambda}^{\text{QFT}} / \rho_{\Lambda}^0$ between the theoretical QFT computation of the vacuum energy density and its presently observed value, $\rho_{\Lambda}^0 \sim 10^{-47} \text{ GeV}^4$, obtained from modern cosmological data [36]. For instance, the Standard Model contribution from electroweak interactions generates the huge ratio $r \sim 10^{55}$ [16]. If that is not enough, there is another pressing CC problem, the “coincidence problem”; namely, to understand why ρ_{Λ}^0 is precisely of the same order of magnitude as the current matter density ρ_m^0 . Observations indeed provide $\rho_{\Lambda}^0 / \rho_m^0 = \Omega_{\Lambda}^0 / \Omega_m^0 \simeq 7/3 = \mathcal{O}(1)$. This is of course very puzzling because ρ_m decays fast as $\rho_m \sim 1/a^3$ at any epoch. So, why on earth is ρ_m almost equal to ρ_{Λ} at our living epoch (i.e. at $a = 1$)?

It is generally accepted that the coincidence problem could be ameliorated if the DE would be a dynamical quantity, e.g. related to some scalar field. Alternatively, it could be that the CC term is not really constant and varies together with some cosmic parameter. At the moment, only phenomenological models are available on all these options [37,38]. Most definitely, a more fundamental possibility would be that the CC were a dynamical variable tied to genuine quantum effects. In such case, the RG should be the most appropriate theoretical instrument to explore this framework [7–13,15,16] and elucidate its phenomenological consequences [17–22], including its possible connection with the coincidence problem [39].

A fundamental equation playing an important role here is the covariant energy conservation. In the Friedmann–Lemaître–Robertson–Walker (FLRW) cosmological context, the Bianchi identity fulfilled by the Einstein’s tensor on the *l.h.s.* of Eq. (2) implies the following relation among the various terms on its *r.h.s.*:

$$\frac{d}{dt} [G(\rho_{\Lambda} + \rho_m)] + 3GH(\rho_m + p_m) = 0, \quad (7)$$

where $H = \dot{a}/a$ is the expansion rate or Hubble function. This equation does not rule out neither H -dependence nor a -dependence of the CC, which can take place either because of the energy exchange between vacuum and matter or due to the variable Newton “constant” G [19,20,22]. In both cases, there is the possibility for a time-evolving cosmological term, $\dot{\rho}_{\Lambda} \neq 0$, which can be a natural alternative to the dynamical DE models exclusively based on ad hoc scalar fields [37].

The above consideration shows that the way leading to CC running is, in principle, open and perfectly consistent with covariance. However, are the quantum effects ultimately responsible for the CC dynamics? At present, our knowledge of QFT in curved space–time is not sufficient to give a definite answer to this question [16,31]. There is, however, a strong hint in favor of the existence (despite the complicated form) of the quantum corrections under discussion. All kinds of such corrections can be seen as contributions to the effective action of gravity, where the CC plays a central role at low energies. Therefore, they can be considered in the framework of the induced gravity paradigm [40]. It is well known that the induced gravitational action *always* has a one-parameter ambiguity [30,40]. The parameter behind this ambiguity must have some physical sense and, in the cosmological setting, it may be related to $a(t)$, $H(t)$ or some combination. This relation opens the door to the physically relevant RG running of the various terms in the vacuum action, including the CC.

3. What means RG running in QFT and in cosmology

Conventionally, the classical theory starts from establishing its action S . The quantum theory, instead, is often characterized by the S -matrix elements. However, in the QFT framework, it is also advantageous to use the effective action (EA) Γ of the mean fields (see, e.g., [41,42]), which can be looked upon as a generalization of the classical action in the quantum domain. This approach becomes especially significant in the presence of gravity, where the definition of the S -matrix can be problematic. Taking variations of the EA, one arrives at the Green functions and finally to the amplitudes. At this point it is important to remember that, in contrast to the classical action, the EA always has certain ambiguities, which eventually disappear in the amplitudes. These ambiguities have a manifold origin: they may come e.g. from the choice of the parametrization (or gauge fixing, as a particular case) of the quantum fields; or from the choice of the renormalization scheme; and also from the dependence of the various parts of the EA on the renormalization parameter μ . Indeed, the latter can be a rather artificial quantity, as e.g. in the case of the MS scheme, but in some cases it can be chosen more physically and represent e.g. an arbitrary subtraction point in momentum space [41].

The derivation of the EA and working out its ambiguities can be regarded as the main target of QFT. In general, one cannot completely calculate the EA, which is typically given by a non-local and non-polynomial expression in the mean fields. Quite often there is a special sector of the EA (viz. the one containing the leading quantum effects) which can be easily accounted for because these effects become just parameterized by the aforementioned arbitrary mass scale μ . The appearance of this scale is characteristic of the renormalization procedure in QFT owing to the intrinsic breaking of scale invariance by the quantum effects. The μ -dependence shows up in many places in the EA, and in particular also in certain non-local parts related to divergences. Despite the neat cancellation of the overall μ -dependence in the EA, the different quantum parts are parameterized by μ , and this apparently innocent fact is absolutely crucial as it enables us to restore the structure of form factors, namely the pieces that transport the physical information of the quantum effects. E.g., in the one-loop QED with zero masses (or, equivalently, at very high energy), the electromagnetic part of the EA has the form

$$\Gamma_{\text{em}}^{(1)} = -\frac{1}{4e^2(\mu)} \int d^4x F_{\mu\nu} \left[1 - \frac{e^2(\mu)}{12\pi} \ln \left(-\frac{\square}{\mu^2} \right) \right] F^{\mu\nu}, \quad (8)$$

where $e(\mu)$ is the renormalized QED charge in the MS scheme. It is apparent from (8) that the effective (or “running”) QED charge

in momentum space satisfies

$$\frac{1}{e^2(Q^2)} = \frac{1}{e^2(\mu^2)} - \frac{1}{12\pi^2} \ln \left(\frac{|Q^2|}{\mu^2} \right) \quad (|Q^2| \gg m_e^2). \quad (9)$$

This relation also follows upon integrating the differential equation

$$\mu \frac{de(\mu)}{d\mu} = \frac{e^3}{12\pi^2} \equiv \beta_e^{(1)} \quad (10)$$

from μ to μ' and then replacing $\mu'^2 \rightarrow |Q^2|$. Here $\beta_e^{(1)}$ is the β -function of QED at one-loop in the MS. Eq. (10) is correspondingly called the RG equation of the renormalized charge in the MS scheme. Notice from (9) that $e(Q^2)$ increases with $|Q^2|$, as expected from the non-asymptotically free character of QED – a well tested feature of this theory.

The remarkable property of the μ -dependence is that the high-energy limit of the theory is reproduced, in the leading approximation, by taking $\mu \rightarrow \infty$. Due to the simplicity of the form factor for the massless case (the above expressions are a nice illustration) one can always restore such form factor using the μ -dependence. In other words, using this “RG-trick” one can immediately retrieve the non-trivial structure of the form factor as a function of the \square -operator. In momentum space, the (artificial) μ -dependence paves our way to discover the explicit (and physical) Q -dependence (9). In fact, this is the essential and practical aspect of the RG-method in Particle Physics: being the combination Q^2/μ^2 the natural variable in the renormalized scattering amplitudes, the RG helps us to find out physical quantum effects that “run” with the energy Q (or some external field) by just inspecting the μ -parameterization inherent to the various parts of the S -matrix and EA.

The simple QED example above can be generalized for any parameter in QFT, as is well known from standard RG arguments [41]. In particular, it also applies to the CC in flat space as far as the μ -dependence is concerned [42]. However, whereas the transition from (10) to the physical Q -running (9) is well established in Particle Physics, the situation with the CC is more delicate. To start with, if the RG method can be applied in cosmology, it should mean that the vacuum energy density becomes a running parameter. Therefore, as in the previous case (10), there should exist a fundamental RG equation of the form

$$\mu \frac{d\rho_\Lambda}{d\mu} = \beta_\Lambda(P, \mu), \quad (11)$$

which is supposed to describe the leading quantum contributions to it, where β_Λ is a function of the parameters P of the effective action (EA). The quantity ρ_Λ in (11) is a (μ -dependent) renormalized part of the complete QFT structure of the vacuum energy. While the full vacuum energy is a physical observable, and hence overall μ -independent, those parts of it related to quantum effects are still parameterized by μ and hence may contain relevant information on the physical running of the form factors (similar to the QED example mentioned above), except that here the correspondence of μ with a physical quantity (Q in the QED case) is not transparent in the cosmological setting. We have said that the RG technique can actually be extended to the whole Particle Physics domain; yet, not even in this case can one arbitrarily choose a subtraction scheme for a particular calculation. For instance, in QCD one cannot choose the on-shell scheme (quarks are never free particles on the mass shell), and so here we must content ourselves with unphysical off-shell schemes, such as the MS. But this is no obstacle for computing physically meaningful observables at the level of amplitudes and cross-sections, basically because the UV recipe $\mu \rightarrow Q$ is absolutely unambiguous and robust.

In cosmology the situation is more complicated, and the final contact with physics is correspondingly more subtle. The root of

the difficulty is partly because (as remarked above) the physical scale behind the quantum effects is not obvious. Moreover, when addressing the cosmology of our present Universe, we are actually dealing with the infrared domain, and hence one is unavoidably led to consider the massive case and the decoupling effects. Obviously this cannot be automatically handled in the MS scheme, as the MS violates the decoupling theorem [43]. The simplest option is to proceed as in QCD, namely to assume a “sharp cut-off”, which means to disregard completely the contributions of massive fields at the energy scale below their mass and, at the same time, treat their contributions above the proper mass scale as high-energy ones, without taking decoupling into account. This was exactly the option adopted in [15].

A crucial question is: what is the relevant physical quantity playing the role of Q in cosmology? The answer is neither obvious nor unique, as there are several reasonable candidates. Yet, it must be linked to the existence of a non-trivial external metric background [12,13,15,16]. For instance, the dynamical properties of this curved background (viz. the expanding FLRW space-time) can be characterized by the expansion rate H . If so, they are expected to induce a functional dependence $\rho_\Lambda = \rho_\Lambda(H)$ in which, as usual, the quantum effects should be parameterized by some renormalization scale μ . Lacking, however, at present of the suitable techniques to tackle a full-fledged computation of the decoupling effects, one can at least take the general covariance as the main guide. Overall, it suggests that the solution of the RG equation (11) should lead to an even-power law of the Hubble expansion rate:

$$\rho_\Lambda(H) = \rho_\Lambda^0 + \frac{3\nu}{8\pi} M_P^2 (H^2 - H_0^2) + \mathcal{O}(H^4), \quad (12)$$

where ρ_Λ^0 and H_0 are the current values of the CC and the Hubble rate, respectively, and ν is a coefficient playing the role of β -function. It is of course understood that the $\mathcal{O}(H^4)$ effects are negligible in the current Universe. This expression is precisely of the “soft-decoupling” form first introduced in [16]. The above expression is the kind of running law that has been tested and further elaborated in many subsequent papers on the CC running [17–23,44].

Let us summarize the general features which are indispensable for a proper understanding and correct use of the RG. Even though this is standard, it is nevertheless necessary, since it is a source of considerable confusion in the recent literature, specially when applied to cosmology [25]. The basic doctrine of the RG is that the full renormalized effective action, Γ , whether in the pure realm of QFT, or in the context of cosmology, or for that matter in any other conceivable context of theoretical physics, must not depend on the numerical value of the arbitrary renormalization scale μ . Moreover, this is true in the MS scheme or in any other subtraction scheme (e.g. the momentum subtraction scheme, which is more physical and closer to the on-shell scheme). This property belongs to the very fundamental definition of the renormalized EA, which must always be equal to the bare EA. Let us e.g. assume dimensional regularization (with n space-time dimensions) and let Φ and P be the full sets of fields and parameters, respectively. If we denote the bare quantities with a subindex 0, we have

$$\begin{aligned} \Gamma_0[g_{\alpha\beta}, \Phi_0, P_0, n] &= \Gamma[g_{\alpha\beta}, \Phi, P, n, \mu] \\ \iff \mu \frac{d\Gamma}{d\mu} &= 0. \end{aligned} \quad (13)$$

This relation holds in both flat and curved space-time and for all kinds of renormalizable theories. Here the μ -dependence appears in order to restore the canonical dimensions of the renormalized

fields and parameters on the *r.h.s.* of (13), and manifests itself in the following two ways:

- 1) All renormalized fields Φ and parameters P depend on μ : in Eq. (13) it is implicitly understood that $\Phi = \Phi(\mu)$ and $P = P(\mu)$.
- 2) The functional form of EA depends explicitly on μ .

These two μ -dependencies always cancel perfectly, for otherwise the renormalized EA would be overall μ -dependent, violating its own definition (13). So, if we take together the two types of μ -dependencies, we just eliminate entirely the possibility to use the RG as a tool to explore the structure of the quantum corrections. The use of the RG requires, therefore, that the implicit and explicit μ -dependencies 1) and 2) are used separately.

In particular, in the MS, being μ an artificial mass unit, its practical use always requires an additional effort. In the flat space theory we meet two different standard interpretations of μ , each of them implying some relation between MS and other, more physical, renormalization schemes in the corresponding limit. In the case 1) above we have to look for the correspondence with the momentum subtraction scheme at high energies. The case 2), instead, is mainly used for the analysis of phase transitions, meaning that we associate μ with the mean value of the almost static (usually scalar) field and arrive at the interpretation of the μ -dependent effective potential (see next section).

The use of μ is justified only if we are clearly understanding which physical parameter is behind it. For instance, in the case of (8) we associate μ with the value of a typical momentum Q in the scattering amplitudes and arrive at the Q -dependence of the coupling constant, see Eq. (9). Similar dependence leads to the asymptotic freedom in the non-Abelian gauge theories. Obviously, the correspondence of μ with Q (or any other physical parameter) can be performed *only if* the μ -dependence is retained in the running parameters, but it would be impossible if one would eliminate μ by canceling it with the other (explicit) μ -dependencies associated to the functional dependence of the EA!

At present, the use of the MS-based RG for the CC and, in general, for the gravitational applications, is fairly non-trivial. The reason is simple but categorical: in the presence of a non-vanishing CC, the space-time cannot be flat, and therefore the calculations cannot be performed by expanding around a flat background (which is the only technique we know to perform calculations in practice [45]). In this situation, the μ -dependence can be hardly associated to some scattering amplitudes because the S -matrix cannot be straightforwardly defined for non-flat spaces. Furthermore, except for the simplest static spaces, the definition of the effective potential is not straightforward in curved spaces [46]. Finally, in curved space the almost static scalar field does not mean, in general, to have static metric or static curvature.

4. Effective potential and running Λ

The effective potential [41], being the quantum analog of the classical potential, appears as the natural tool to explore the vacuum energy in QFT, and one may suspect that its properties have an important bearing on the CC problem. However, to understand the precise connection between the two concepts is not so simple as one might naively think. Therefore, it seems necessary to clarify this point in order to avoid misleading conclusions [25]. Let us consider the RG equation for the effective potential V of the real massive scalar field with the $\lambda\phi^4$ interaction. The CC can be included into the scalar potential in the form of an additive hm^4 term, where m is the scalar mass and h is an independent dimensionless parameter. The RG equation for V follows from the fact

that, for constant mean fields and in the local approximation, the effective action boils down to $\Gamma = -V\Omega$, where Ω is the space-time volume. The RG-invariance of Γ , expressed by Eq. (13), leads to the following PDE for the renormalized effective potential:

$$\left(\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial \lambda} + \gamma_m m^2 \frac{\partial}{\partial m^2} + \gamma_\phi \phi \frac{\partial}{\partial \phi} + \beta_h \frac{\partial}{\partial h} \right) \times V(\phi, m^2, \lambda, h, \mu) = 0. \quad (14)$$

It would be incorrect to think of this equation as being tantamount to a kind of non-running theorem for the vacuum energy and the CC. This way of thinking could only be the result of a misleading interpretation [25] about the significance of the RG and of the notion of vacuum energy in the presence of gravity. Indeed, neither the effective potential nor its vacuum expectation value can be identified with the vacuum energy in the cosmological context (cf. Section 5). Put another way: in flat space, V cannot provide by itself any crucial insight on the possible running of the CC. Yet, this does not mean that we cannot extract valuable information using the RG technique if we, first of all, embed this theory in a non-trivial external background [15,16]. Let us, thus, analyze the situation more carefully, step by step:

- First of all a technical point: Eq. (14) is, in fact, a sum of the two independent equations, to wit, one can split the overall effective potential as a sum of two pieces, the ϕ -independent (vacuum) term and the ϕ -dependent (scalar) term, as follows:

$$V(\phi, m^2, \lambda, h, \mu) = V_{\text{scal}}(\phi, m^2, \lambda, \mu) + V_{\text{vac}}(m^2, \lambda, h, \mu). \quad (15)$$

In this way, we can also split the RG equation (14) into two independent RG identities:

$$\begin{aligned} & \left(\mu \frac{\partial}{\partial \mu} + \beta_\lambda \frac{\partial}{\partial \lambda} + \gamma_m m^2 \frac{\partial}{\partial m^2} + \gamma_\phi \phi \frac{\partial}{\partial \phi} \right) \times V_{\text{scal}}(\phi, m^2, \lambda, \mu) = 0, \\ & \left(\mu \frac{\partial}{\partial \mu} + \beta_\lambda \frac{\partial}{\partial \lambda} + \gamma_m m^2 \frac{\partial}{\partial m^2} + \beta_h \frac{\partial}{\partial h} \right) \times V_{\text{vac}}(m^2, \lambda, h, \mu) = 0. \end{aligned} \quad (16)$$

In order to understand the origin of this splitting, one has to introduce the functional called effective action of vacuum Γ_{vac} . It is that part of the overall EA which is left when the mean scalar field ϕ is set to zero: $\Gamma_{\text{vac}} = \Gamma[\phi = 0]$. Thus, it is a pure quantum object which only depends on the set of parameters $P = m, \lambda, \dots$ of the classical theory. At the functional level,

$$e^{i\Gamma_{\text{vac}}} = \int \mathcal{D}\phi e^{iS[\phi; J=0]}, \quad (18)$$

where the source J is set to zero. In this way, the functional Γ_{vac} is the generator of the proper vacuum-to-vacuum diagrams. In flat space, these diagrams are removed by normalizing the functional to one. But in curved space they are significant. From the RG-invariance of the renormalized EA – see Eq. (13) – it follows immediately the μ -independence of the renormalized functional Γ_{vac} and, therefore, we arrive at the second identity (17) for the vacuum part of the effective potential, while the first identity is the result of the subtraction of (17) from (14). The net result is that the vacuum and matter parts of the effective potential are overall μ -independent separately and no cancelation between them can be expected.

- The μ -independence of the EA holds by construction and definitely does not mean that there is no physical running, as it was assumed in [25]. As we have already discussed in Section 3, and as reflected by Eq. (14), the explicit μ -dependence of the function $V(\phi, m^2, \lambda, h, \mu)$ is automatically canceled by the μ -dependencies

of the field ϕ and parameters λ, m, h . Using both dependencies at the same time makes no sense, just because we know in advance that they cancel. This is true for *any* renormalizable theory and for *any* sector of such theory, in particular for the NJL model, QED and QCD. Recall e.g. that for the NJL model the renormalization can be simply linked with the CC case just by changing the parametrization of the external metric, see Eq. (6).

- In the vacuum sector, the one-loop effects modify the relation (4) as follows:

$$V_{\text{vac}} = \rho_{\Lambda}^{\text{vac}}(\mu) + \delta\rho_{\Lambda}^{\text{vac}} + \bar{V}_{\text{vac}}^{(1)}. \quad (19)$$

For a scalar field with mass m , the dimensionally regularized form of the vacuum-to-vacuum diagram in flat space gives

$$\begin{aligned} \bar{V}_{\text{vac}}^{(1)} &= \frac{1}{2} \mu^{4-n} \int \frac{d^{n-1}k}{(2\pi)^{n-1}} \sqrt{k^2 + m^2} \\ &= \frac{1}{2} \beta_{\Lambda}^{(1)} \left(-\frac{2}{4-n} - \ln \frac{4\pi\mu^2}{m^2} + \gamma_E - \frac{3}{2} \right), \end{aligned} \quad (20)$$

where $n \rightarrow 4$ in the last expression. The one-loop β -function of the vacuum Λ term reads [16]

$$\beta_{\Lambda}^{(1)} = \frac{m^4}{32\pi^2}. \quad (21)$$

Eq. (20) is divergent and needs a subtraction. If we adopt the $\overline{\text{MS}}$ subtraction scheme, the counter-term $\delta\rho_{\Lambda}^{\text{vac}}$ gets fixed in such a way that the renormalized result is

$$V_{\text{vac}} = \rho_{\Lambda}^{\text{vac}}(\mu) + \bar{V}_{\text{vac}}^{(1)} = \rho_{\Lambda}^{\text{vac}}(\mu) + \frac{1}{2} \beta_{\Lambda}^{(1)} \left(\ln \frac{m^2}{\mu^2} - \frac{3}{2} \right). \quad (22)$$

Plugging the β -function (21) on the *r.h.s.* of the RG equation (11), we can easily integrate it (because at one-loop the mass m does not run with μ):

$$\rho_{\Lambda}^{\text{vac}}(\mu) = \rho_{\Lambda}^{\text{vac}}(\mu_0) - \frac{1}{2} \beta_{\Lambda}^{(1)} \ln \frac{\mu_0^2}{\mu^2}. \quad (23)$$

One can check that, after replacing Eq. (23) into Eq. (22), we arrive at the expression in which μ is replaced by μ_0 . This is the realization, in this particular example, of the RG-invariance of the vacuum part of the effective potential, i.e. of Eq. (17). The procedure can be easily extended to any loop order and $V_{\text{vac}}(m, \lambda)$ is in general a function of both the mass m and of the self-coupling λ .

- The situation for the induced vacuum energy density is similar. The form of the effective potential V for the scalar field with the classical potential $U(\phi)$ is well known (see, e.g., [41]):

$$\begin{aligned} V(\phi) &= U(\phi) + \bar{V}^{(1)}(\phi) \\ &= U(\phi) + \frac{1}{64\pi^2} U''^2 \left(\ln \frac{U''^2}{\mu_0^2} - \frac{1}{2} \right). \end{aligned} \quad (24)$$

In the case of

$$U(\phi) = \frac{m^2 \phi^2}{2} + \frac{\lambda \phi^4}{4!}, \quad (25)$$

the one-loop correction yields

$$\bar{V}^{(1)}(\phi) = \frac{1}{64\pi^2} \left(m^2 + \frac{\lambda \phi^2}{2} \right)^2 \left[\ln \frac{(m^2 + \lambda \phi^2/2)}{\mu^2} - \frac{3}{2} \right]. \quad (26)$$

To obtain $V_{\text{scal}}(\phi, m^2, \lambda, \mu)$, we just subtract from (24) the vacuum part at one-loop, which is given by the second term on the *r.h.s.* of (22) – and also by the value of $V(\phi)$ at $\phi = 0$, Eq. (22). The ϕ -dependent part of the potential $V_{\text{scal}}(\phi, m^2, \lambda, \mu)$ is just given

by the difference $V(\phi) - V(\phi = 0)$. The fact that it depends explicitly on μ and, at the same time, satisfies (17) is because there is still the implicit μ -dependence that is associated to the renormalization of the parameters λ and m . Once the two types of μ -dependencies meet together, they annihilate each other.

In order to give one more illustrative example, let us concentrate on the massless case. The RG equation for λ has a form similar to (10) in the massless QED case:

$$\mu \frac{d\lambda}{d\mu} \Big|_{n \rightarrow 4} = \beta_\lambda^{(1)} = \frac{3\lambda^2}{(4\pi)^2}, \quad \lambda(\mu_0) = \lambda_0. \quad (27)$$

To first order in λ , the solution of (27) is given by $\lambda(\mu) = \lambda_0 + \beta_\lambda^{(1)} \ln(\mu/\mu_0)$. The obtained result is just the physical running coupling $\lambda = \lambda(\mu)$ after $\mu \rightarrow |Q|$. Inserting it in (26) and disregarding the $\mathcal{O}(\lambda^3)$ terms, one obtains a similar expression for V in which μ is traded for μ_0 . In other words, this confirms that V_{scal} is RG-invariant to the order under consideration, as we expected. On the other hand, the physically relevant ϕ -dependence is not affected by our manipulations, also as expected. Similar considerations can be done for the massive case.

What is the lesson from this simple example? It is the following: in order to gather some physical insight from the μ -dependence, we should not use Eqs. (26) and (27) at the same time, but separately. No physical information can be extracted from the cancelation of μ -dependence in any sector of the EA, except to confirm the consistency of the overall RG procedure. In contrast, if one keeps the μ -dependencies separately, then, in the first case, this dependence is useful to derive the form of effective potential for the massless field, whereas in the second case it corresponds to the momentum-dependence in the scattering amplitudes at high energies.

- An essential point in the discussion of the cosmological vacuum energy is the presence of an external metric (the FLRW one in the standard cosmological scenario). This presence is tremendously crucial for the RG applied to CC, however it was not taken into account in the considerations presented in [25].

In flat space, the one-loop vacuum contributions are given by just closed loops of matter fields without external legs. It corresponds to the V_{vac} piece in Eq. (15), and at one-loop simply reads (22). At any order, such loop contributions do not depend on any physical quantity apart from the masses and couplings of the matter particles. So, the μ -dependence, although mathematically present, has no physical implication. The situation changes dramatically in curved space-time, where the relevant Feynman diagrams include matter field loops with a number of external legs of the metric [31]. In this situation, there are physically reasonable “identifications” of μ [15–19].

- For example, in the approach used in [15,16], the MS-based RG was employed for all parameters, including CC, λ and m . Here μ was identified with the H -dependent sharp cut-off $\mu \sim \rho_c^{1/4} \sim \sqrt{H M_P}$. Therefore, the contributions from particles with masses $m > \sqrt{H M_P}$ were excluded from the running of λ , m and CC. As we have discussed, the choice of the cut-off is not unique because the status of the renormalization technique in this context is not perfect at present, but the use of the RG was consistent and technically correct. Similarly, we arrived at (12) through the alternative ansatz $\mu = H$, and this is also technically correct within the corresponding subtraction scheme (in fact, one which is conceptually more related to momentum subtraction). See also [12,13] for similar identifications.

- Although the physical vacuum energy is independent of μ , the reason for choosing (“identifying”) a particular value is because we do not know the full quantum structure of the vacuum energy in curved space-time. Therefore, one can estimate it by picking a

representative energy parameter of the system (in this case H , the expansion rate of the Universe). The procedure is similar to computing a QCD cross-section at a given order, say $\sigma(e^+e^- \rightarrow q\bar{q})$ at order $\mathcal{O}(\alpha_{\text{em}}^2 \alpha_s^2)$ in the electromagnetic and strong coupling constants $\alpha_i = g_i^2/4\pi$. The result is explicitly depending on μ since, at this order, one must renormalize the strong coupling constant $\alpha_s(\mu^2)$. The μ -dependence would disappear, of course, if we could compute the cross-section to all orders. However, to get a numerical estimate at a finite order, the value of μ is typically “identified” with the energy/momentum of the process ($\mu \rightarrow |Q|$), as this avoids the effect from large logs. In the cosmological case, the situation is more complicated; in part because the explicit computation is not feasible in an intrinsically curved space-time background with $\Lambda \neq 0$ (not even at the one-loop level), and in part also because choosing μ is more subtle when we are working in the infrared regime.

- Last, but not least, in curved space-time the minimally interacting scalar field is not renormalizable (see, e.g., [30]) and hence the MS-based RG cannot be applied as it is done in [25]. In the presence of the non-minimal scalar-curvature term $R\phi^2$, the SSB becomes quite non-trivial and the renormalization of the theory is more complicated [46].

In summary, a necessary condition for the physical running of the CC is to have a non-trivial gravitational background. However, we do not know if such condition is sufficient. Hence, in order to establish (or disprove) such running one has to derive explicitly the quantum corrections, or at least indicate the form that they can have. Since we cannot perform the explicit calculations for the time being, in the next section we address only the possible form of the quantum effects.

5. Possible forms of the quantum effects on the vacuum action in curved space-time

The possible form of the quantum corrections to the Einstein–Hilbert action and to the CC term should be represented by some functional of the metric and its derivatives, while the other fields are in the stable vacuum states. Therefore, for the present consideration we do not need to distinguish between the vacuum and induced parts of the quantum corrections. Within QFT in curved space, the classical action of vacuum has the form (1) without the matter term, where

$$S_{\text{HD}} = \int d^4x \sqrt{-g} \{a_1 C^2 + a_2 E + a_3 \square R + a_4 R^2\}. \quad (28)$$

Here $C^2 = R_{\mu\nu\alpha\beta}^2 - 2R_{\alpha\beta}^2 + (1/3)R^2$ is the square of the Weyl tensor and $E = R_{\mu\nu\alpha\beta}^2 - 4R_{\alpha\beta}^2 + R^2$ is the integrand of the Gauss–Bonnet topological term.

What could be the form of the quantum corrections to (28)? The first option is to perform explicit calculations of the quantum corrections, including the finite part. The history of such calculations goes back to the early works on quantum theory in curved space [47]. The most complete result in this direction has been obtained in [45] through the derivation of the Feynman diagrams in the framework of linearized gravity and also by using the heat kernel solution of [48]. The output of such calculation includes only the terms of zero, first and second order in the curvature tensor. For example, the one-loop result for the massive real scalar field reads

$$\bar{\Gamma}_{\text{vac}}^{(1)} = \frac{1}{2(4\pi)^2} \int d^4x \sqrt{-g} \times \left\{ \frac{m^4}{2} \cdot \left(\frac{1}{\epsilon} + \frac{3}{2} \right) + \left(\xi - \frac{1}{6} \right) m^2 R \left(\frac{1}{\epsilon} + 1 \right) \right\}$$

$$+ \frac{1}{2} C_{\mu\nu\alpha\beta} \left[\frac{1}{60\epsilon} + k_W(a) \right] C^{\mu\nu\alpha\beta} \\ + R \left[\frac{1}{2\epsilon} \left(\xi - \frac{1}{6} \right)^2 + k_R(a) \right] R \}. \quad (29)$$

Here $1/\epsilon \equiv 2/(4-n) + \ln(4\pi\mu^2/m^2) - \gamma_E$ is a parameter related to dimensional regularization, and ξ is the coefficient of the non-minimal coupling. It is clear that the first term on the r.h.s. of (29) coincides with the flat-space expression in Eq. (20). The full result (29), therefore, provides the corresponding generalization for the case when there is a non-trivial background. The non-local form factors have the form

$$k_W(a) = \frac{8A}{15a^4} + \frac{2}{45a^2} + \frac{1}{150}, \\ k_R(a) = A\tilde{\xi}^2 + \left(\frac{2A}{3a^2} - \frac{A}{6} + \frac{1}{18} \right) \tilde{\xi} + \frac{A}{9a^4} - \frac{A}{18a^2} + \frac{A}{144} \\ + \frac{1}{108a^2} - \frac{7}{2160}, \quad (30)$$

where we used notations

$$\tilde{\xi} = \xi - \frac{1}{6}, \quad A = 1 - \frac{1}{a} \ln \left(\frac{2+a}{2-a} \right), \quad a^2 = \frac{4\Box}{\Box - 4m^2}. \quad (31)$$

Let us note that similar expressions have been obtained for massive fermion and vector cases [45] and also for the scalar field background, where the form factors were calculated for the $\phi^2 R$ and ϕ^4 terms [49]. Looking at these expressions, it is clear that it is no longer possible to use an effective potential approach as in the flat space case considered in Section 4, since some of the new terms involve non-local contributions and are explicitly dependent on the external momenta, including in the gravitational sector. In the UV limit the non-local form factors have the logarithmic behavior similar to (8), while in the IR limit they follow the Appelquist and Carazzone theorem [43].

The form factors (30) contain important information about quantum corrections. However, the CC and Einstein–Hilbert sectors of (29) carry no relevant form factors. In both cases there are divergences, and the μ -dependence is identical to the one of the MS-based RG analysis that we have presented in Section 4. However, there is nothing real behind this formal μ -dependence [42]. Contrary to the higher derivative sectors, we do not observe physical running in the CC and G sectors of the vacuum action. It is noticeable that, in these two cases, there is no obvious correspondence between the RG approaches in the MS and physical subtraction schemes, not only in the IR, but also in UV. The origin for this lack of correspondence is that, when the CC is present, the space-time background is unavoidably curved; hence, the expansion around the flat background, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, although computationally feasible, it does miss part of the most relevant quantum corrections. The passing to the flat space does not affect the form factors of the higher derivative terms, but the ones for the CC and Einstein–Hilbert terms just vanish, because $\Box\Lambda = 0$ and $\Box R$ is an irrelevant total derivative term. Hopefully, the problem can be solved by performing calculations on some dynamical or at least de Sitter background, but even in such relatively simple case it has not been done yet. Such calculations look quite non-trivial, because one needs to compute not only the divergences, but also the first finite non-local correction using some (yet unknown) physical renormalization scheme.

Coming back to the discussion on the viability of the CC running, we can prove a “no-go” theorem concerning the possible/impossible form of the relevant quantum corrections to the CC term. We can show that these quantum effects cannot just be

given by non-local terms, if they are still polynomial in the curvatures. In order to support this statement, let us consider some polynomial term in curvatures having a finite number of Green functions insertions. These insertions should correspond to the massive quantum field propagator in curved space, but since the polynomial term admits the flat space expansion, we can take the $\mathcal{O}(h_{\mu\nu})$ term for each curvature and flat-space propagator. The simplest possible terms of this sort are

$$R_{\mu\nu} \frac{1}{\Box + m^2} R^{\mu\nu}, \quad R \frac{1}{\Box + m^2} R, \quad (32)$$

and the other possible terms differ from these ones by a finite number of curvature terms and corresponding Green functions. Inasmuch as we are always interested in the large mass limit, we can expand the propagator as

$$\frac{1}{\Box + m^2} \sim \frac{1}{m^2} \left(1 - \frac{\Box}{m^2} + \dots \right). \quad (33)$$

The first term in the parenthesis just gives a contribution to the local terms in the vacuum action (1), whereas the second term generates a $\mathcal{O}(H^6/m^4)$ contribution and is phenomenologically irrelevant (notice that $\Box \sim H^2$ and $R \sim H^2$ in the FLRW metric). Furthermore, the higher order curvature terms give even smaller contributions. The upshot is that we have no relevant running of CC and $1/G$ from the polynomial non-local terms.

Does it mean that the running is impossible? No. Let us remember that the relevant Feynman diagrams renormalizing the CC term consist of closed loops of matter particles with an unrestricted number of external gravitational legs and a corresponding infinite number of Green function insertions of the matter fields. In other words, one actually generates a series of infinite non-local products of curvatures. The actual renormalization effect on the CC should come from the resummation of this series, and the result is expected to depend on a massless Green function, say $G_0 \sim 1/\Box$. For instance, let us take the form $R\mathcal{F}(G_0R)$, for some unknown function \mathcal{F} of dimension 2. The canonical possibility would be $\mathcal{F} = M^2 G_0 R$, where M is an effective mass. It is easy to see that, in this case, the resummed expression would be of order $M^2 H^2$. Therefore, if M is not far away from M_P , it would lead to the kind of (potentially measurable) running law (12) which we already expected from general considerations of covariance [16].

The positive sign that this kind of resummation is feasible is that, in the unique case when we can find an exact solution for the EA in curved space-time, that is to say, for the anomaly-induced action (34), we do observe a very similar sort of resummation into a massless propagator. The anomaly-induced action (see, e.g., [31] and references therein) has no sign of the Green functions of the original quantum fields, but it leads to the fourth order conformal operator $\Delta_4 = \nabla^4 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\nabla^2 + \frac{1}{3}(\nabla^\mu R)\nabla_\mu$, which was not present in the initial theory.

An indirect confirmation, through explicit calculation, that the RG-like corrections to the CC and Einstein–Hilbert terms are possible, can be obtained within the method suggested in [50,51] (see also [22]). We refer to the mentioned papers for details and just present the final result at the one-loop level:

$$\bar{\Gamma} = - \int d^4x \sqrt{-\bar{g}} e^{2\sigma} [\bar{R} + 6(\bar{\nabla}\sigma)^2] \cdot \left[\frac{1}{16\pi G} - f \cdot \sigma \right] \\ - \int d^4x \sqrt{-\bar{g}} e^{4\sigma} \cdot \left[\frac{\Lambda}{8\pi G} - g \cdot \sigma \right] \\ + \text{higher derivative terms}. \quad (34)$$

The EA (34) is written using a special conformal parametrization of the metric $g_{\mu\nu} = e^{2\sigma(x)} \bar{g}_{\mu\nu}$, similar to Eq. (5). Here g and f are

the MS-based β -functions for the corresponding parameters of the higher derivative action (28). The above expression is a generalization of the RG corrected classical vacuum action (1), in a perfect correspondence with our considerations in Section 3.

Let us stress that the result (34) has been obtained without the polynomial expansion in curvatures and is essentially based on the non-covariant separation of the conformal factor of the metric. As can be seen, in this parametrization the “running” of CC and $1/G$ with σ is perfectly possible. In this sense, this result represents a strong argument in favor of the possibility of the relevant quantum corrections in the CC and Einstein–Hilbert sectors. The reason is that the expression (34) does not depend on the flat-expansion approach and that is why it enables one to reproduce the RG-based result.

6. Conclusions

We have discussed some important theoretical aspects of the CC or vacuum energy density, most particularly the possibility that it can be a running quantity in QFT. In view of the magnificent observational status of cosmology at present, we believe that such discussion is timely and highly convenient to encourage the theoretical community to focus more deeply on the QFT status of the CC term rather than replacing it too often with ad hoc scalar fields and other artificial variables. While the technical limitations of the current QFT methods cannot prove that the CC running takes place, they intriguingly suggest that it is a most natural scenario and that it should be further investigated with renewed efforts. At the same time, these methods cannot disprove such running either. Therefore, all positive statements here must be done with proper caution; and the same is true, of course, for the negative statements. In particular, trying to prove a “no running” theorem on the basis of μ -independence of EA [25] is misleading, because the μ -independence is nothing else but the universal property of EA, which cannot support any constructive statement about the physical running of the CC. The latter, if really exists, should manifest as a functional dependence on the cosmological variables of the expanding background (such as the Hubble rate, scale factor, etc.). The role of μ is merely formal and has no direct physical meaning at all. However, the fact that μ parameterizes the structure of the quantum corrections (as is well known e.g. in QED and QCD) can be the clue to use the RG also in cosmology as the optimal instrument to search for the possible form of the quantum corrections to the vacuum energy and extract their concrete dependence on the physical quantities of the expanding Universe. In our opinion, there is no doubt that this is one of the main challenges for future investigations on the fundamental theoretical aspects of this discipline, and it can be the clue to a most sought-after interface, if not the very touchstone, that may link Particle Physics and Cosmology [16].

Finally, in the present situation, wherein the perspective to obtain a solid QFT result remains still unsettled, it is absolutely legitimate to use the phenomenological approach to investigate the implications of a running CC and to derive the cosmological restrictions on it (see, e.g., the extensive work along these lines presented in Refs. [13] and [15–22]).

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